For example, for the experimental channel described above k = 0.565, $\delta/x|_{x=H} = 0.0385$ and if we take $\varepsilon = 0.05$, we obtain the condition

$$\mathrm{Pe}^*|_{x=H} \geqslant 0.052.$$

NOTATION

δ, H, f(x), height, length, and width of channel; $S(x) = \int_0^x f(x)\,dx$, area of channel wall washed with liquid at distance between channel inlet and cross section x; ω_x, longitudinal velocity component; $\bar{\omega}$, mean velocity over channel section x; $h = \delta/2$, channel half-height; G_m, mass flow rate of liquid; ρ, a, c_p, λ, density, thermal diffusivity, heat capacity, and thermal conductivity of liquid; t, local liquid temperature in channel; t_w, channel wall temperature; t_0, liquid temperature before entrance into channel; $\bar{t}$, liquid mean mass temperature at section x; α, $\bar{\alpha}$, local and mean integral (over surface) heat-liberation coefficients, referenced to temperature difference $(t_w - \bar{t})$; α_0, $\bar{\alpha}_0$, local and mean integral (over surface) heat-liberation coefficients, referenced to temperature difference $(t_w - t_0)$; Nu, $\overline{\mathrm{Nu}}$, Nu_0, Nusselt criteria for heat-liberation coefficients α, $\bar{\alpha}$, $\bar{\alpha}_0$; Pe*, generalized modified Peclet criterion; Pe, Peclet criterion; Nu_{st}, Nusselt criterion for thermally stabilized flow; q_w, density of thermal flux from wall to liquid at section x; Q, thermal flux associated with one channel wall.

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LIQUID-FILM FLOW REGIMES ON A ROTATING SURFACE

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UDC 532.517:532.62

Liquid flow on a rotating disk is analyzed and studied experimentally. Limits of the hydrodynamic regimes are established.

Liquid-film flow on rotating surfaces is employed in thermal mass-exchange devices (evaporators, driers, absorbers, etc.), reactors, and centrifuges in various technological fields. Calculation of such devices presumes a clear understanding of flow hydrodynamics, in particular, of the limits of laminar and turbulent flow.

Despite the large number of studies dedicated to study of the hydrodynamic characteristics of film flow on rotating bodies, only [1-6] indicated the hydrodynamic regime. However, in those studies there are major differences in evaluation of the effects of various parameters (wetting density Γ, angular velocity ω, and surface dimension R) on flow stability. The effect of Γ was considered in [1, 6], while [3] considered only ω and R. In determining the flow regime, Dorfman [8] considered the effects of both angular velocity and initial film thickness δ, considering the latter known. In [4], one of the first studies of film hydrodynamics on rotating surfaces, the authors use an expression for flow-regime determination which includes all the dimensionless quantities used in the works mentioned above [3, 6, 8]:

$$\mathrm{Re}^* = \mathrm{Re}_\Gamma^{-1}\mathrm{Re}_r\,\mathrm{Re}_\delta = \left(\frac{G}{2\pi R\nu\rho}\right)^{-1}\frac{\omega R^2}{\nu}\frac{\omega\delta^2}{\nu}. \tag{1}$$

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It should be noted that this expression cannot serve as a defining criterion, since it contains a liquid-film thickness known beforehand.

On the basis of dimensional analysis, [5] recommended a complex, termed the modified Reynolds number by the authors, that considered the effect of both wetting density and angular velocity on flow regime:

$$\mathrm{Re_m} = \mathrm{Re_f}\ \mathrm{Re_r} = \frac{G}{2\pi R\nu\rho}\ \frac{\omega R^2}{\nu}. \tag{2}$$

Using Eq. (2), [2] determined the flow regime of suspensions in a rotor-film centrifuge. The data of [5] and [2] indicate the critical value of $\mathrm{Re_m}$ to be $0.524 \cdot 10^8$ and $0.585 \cdot 10^8$, respectively.

There are contradictions in the conclusions of the various authors as to the relative location of the laminar and turbulent zones on the rotating surface. If the hydrodynamic mode depends solely on wetting density ($\mathrm{Re_f}$), as in [5, 6], it is quite obvious that the flow will be more turbulized in the direction toward the center of the surface. However, the equations presented in [3, 4] and Eqs. (1), (2) indicate the opposite. In this sense the data of [9] are also contradictory, the author using Eq. (2) to determine hydrodynamic characteristics and then deriving relationships for the mean heat-exchange coefficient with consideration of film flow turbulization in the central portion of the disk, which disagrees with Eq. (2).

None of the published studies offer any explanation of the physical nature of the earlier development of film turbulization on a rotating surface, noted in [2-5], as compared to a fixed surface (for equal $\mathrm{Re_f}$). In this connection it will be useful to rely on certain distinctive peculiarities of film motion under the action of centrifugal forces.

As is well known [6, 17], film motion on a rotating surface is accompanied by intense wave formation. For the case of gravitational film flow, Kapitsa [10] obtained an equation for determination of the film Reynolds number $\mathrm{Re_{f,W}}$ at which wave formation commences. It follows from this equation that with increase in acceleration the value of $\mathrm{Re_{f,W}}$ must decrease. Calculation shows that for other conditions equal, at a centrifugal acceleration of $j = 1000g$ the value of $\mathrm{Re_{f,W}}$ will be almost twice as small as for gravitational motion. Thus, the earlier commencement of wave formation may lead to an earlier (at lower $\mathrm{Re_f}$) flow turbulization.

In our opinion, the hydrodynamics of a film moving on a rotating surface will also be affected by the gaseous medium in contact with the liquid's free surface. It is known [11, 12] that external perturbations acting on the film surface may cause disruption of the stability of a laminar-wave flow at $\mathrm{Re_f}$ values below critical. Among such perturbations are friction at the phase boundary and changes in the gas-flow regime above the film surface [10, 12-16]. Studies [13-16] have shown that pulsations in the gas flow occurring during turbulent flow have a significant effect on film hydrodynamics, encouraging reduction of the critical value of $\mathrm{Re_f}$. It is well known that liquid flow on a rotating disk is accompanied by a companion gas flow, caused by disk rotation. In this case developed turbulent gas flow sets in at $\mathrm{Re_g}$ greater than $(2.7\text{-}3) \cdot 10^5$, while instability commences at $1.8 \cdot 10^5$ [7]. A reduction in the critical value of $\mathrm{Re_g}$ is observed with increase in surface roughness. It must then be noted that the liquid-wave film may be considered [10] as a rough surface with significant protuberance height, inasmuch as the data of [17] indicate that the wave amplitude on a rotating disk comprises 30-50% of the film thickness. Film "roughness" increases with increase in liquid-flow rate (film thickness), which at a given angular velocity leads to decrease in the critical value of $\mathrm{Re_g}$. The turbulent pulsations developing in the gas flow can act on the film, destroying the stability of its flow at lower values of $\mathrm{Re_f}$.

In light of the above considerations, the authors regard it possible to use as the characteristic for transition from laminar-wave flow to turbulent a dimensionless complex which we will term the corrected Reynolds number:

$$\mathrm{Re_c} = \mathrm{Re_f}\ \mathrm{Re_g} = \frac{G}{2\pi R\nu\rho}\ \frac{\omega R^2}{\nu_g}. \tag{3}$$

Equation (3) has a structure similar to Eq. (2), but differs from the latter in considering the effect of gas motion above the film surface. The use of Eq. (3) may also be justified by dimensional analysis.

An experimental study was performed of the hydrodynamics of flow of a film of water, an aqueous solution of glycerin ($\rho = 1.1$ kg/m^3, $\mu = 5.17 \cdot 10^{-3}$ Pa·sec), and a solution of Barf-type surfactants ($\sigma = 36.1 \cdot 10^{-3}$ N/m) under isothermal conditions ($t \simeq 20°$C) on a disk of type Kh18N10T stainless steel with radius $R_{fin} = 0.2$ m rotating in the horizontal plane at angular velocities from 5 to 240 sec^{-1}. Liquid was supplied at the disk center. The disk surface was processed to class 8-9 according to GOST (All-Union State Standard) 2789-59. The

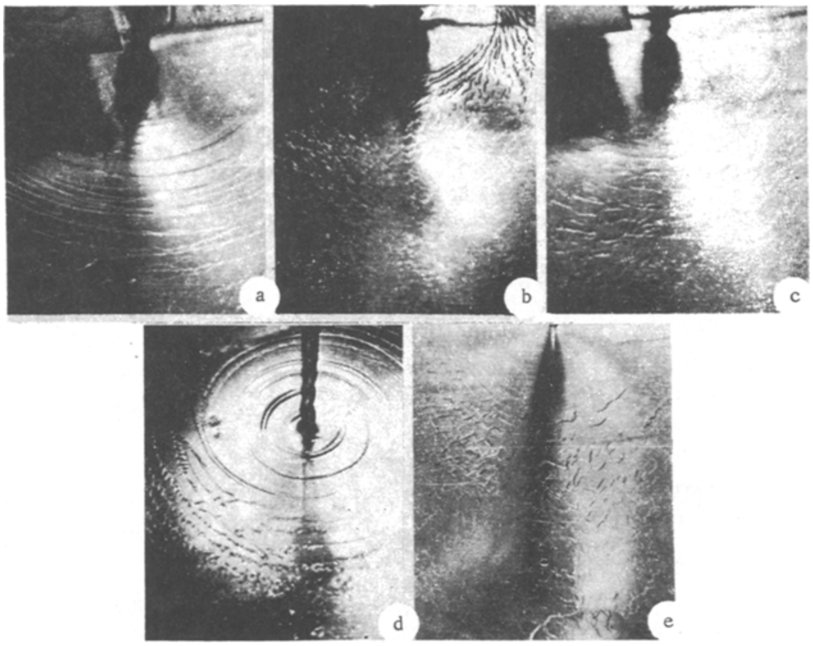

Fig. 1. Effect of disk angular velocity and liquid-flow rate on hydrodynamics of film flow. Glycerin solution: $G = 25 \cdot 10^{-3}$ kg/sec; $Re_f^c = 4.26$: a) $\omega = 73.2$; b) 240 sec^{-1}; $G = 33 \cdot 10^{-3}$ kg/sec; $Re_f^c = 5.62$: c) $\omega = 73.2$; d) 125 sec^{-1}; e) water, $G = 6.5 \cdot 10^{-3}$ kg/sec; $Re_f^c = 5.18$; $\omega = 104/6$ sec^{-1} (incomplete wetting, $R_W = 0.135$ m; $Re_f^W = 4.95$).

centrifugal apparatus was provided with an ST-5 stroboscope of the Tbilipribor factory and synchronized with disk rotation rate so that visual observations and photography of the film surface could be performed. Analysis of measurement errors revealed that under the experimental conditions the maximum relative systematic error did not exceed 14%.

The experimental method ensured constancy of one of the two quantities (ω or G) characterizing the film flow, while the other was varied. The flow character was determined visually and by photography; at times, dye material was introduced into the liquid.

In the experiments with water and the glycerin solution a laminar film with no signs of wave formation was observed over the entire disk only at angular velocities $\omega \leq 41.8$ sec^{-1} and $Re_f^c \leq 2$. In the flow of the surfactant solution a smooth film existed at significantly higher values of Re_f and ω, which indicates an increase in flow stability as in [2]. In the range $\omega \leq 41.8$ sec^{-1} increase in flow rate of water or glycerin to values of $Re_f^c = 20$-25 and of surfactant solution to $Re_f^c = 50$-60 led only to significant wave formation in the central portion of the disk; as the liquid moved over the disk to the periphery the waves gradually damped. With increase in velocity the wave damping ceased and the waves were observed over the entire surface. The intensity and character of wave formation depended on both flow rate and angular velocity (Fig. 1a-d). As is evident from Fig. 1, after the smooth input section concentric waves develop at the center of the disk, it being possible to determine the transition from one wave to the next. The region of concentric wave existence decreased in size with increase in either Re_f or ω. With motion toward the disk edge the waves gradually break down (Fig. 1a,d,e) and transform into solitary waves (in the terminology of [10]) in which the flow picture appears to be that of a series of individual semidroplets (Fig. 1e) rolling over the surface of a film moving in laminar fashion, with smaller waves existing between the larger ones; such waves usually had steep fronts.

At certain values of Re_f and ω the solitary waves gradually lost their form, and beginning at a certain radius a fine-structured disorderly ripple (Fig. 1b,c) could be seen on the liquid surface. The appearance of this rippled region recalls the surface perturbations of a gravitation film flow photographed in [14] with turbulent gas flow above the film. Introduction of dye streaks into the liquid flow revealed that in the ripple region intense washing action occurred, while in the central region the streak flow had a laminar character.

TABLE 1. Effect of Angular Velocity ω and Liquid-Flow Rate G on Transition from Laminar to Turbulent Motion

Liquid studied	ω, sec^{-1}	$G \cdot 10^3$, kg/sec	R_{cr_1}, m	Re_f	$Re_g \cdot 10^{-5}$	$Re_c \cdot 10^{-6}$	$Re_m \cdot 10^{-8}$
Water	240	5,5	0,065	13,5	0,68	0,917	0,855
	200	9,5	0,06*)	25,2	0,478	1,21	1,14
	188	6,5	0,065	15,9	0,502	0,796	0,985
	104,6	6,5	0,16	6,45	1,78	1,15	1,09
Aqueous glycerin solution	240	25	0,075	10,2	0,896	0,806	0,186
	180	25	0,09	8,55	0,967	0,827	0,168
	140	33	0,11	9,2	1,09	1,0	0,218

*Determined by beginning of dye streak diffusion.

The ripple region did not disappear out to the disk edge, but its height changed upon reduction in film thickness. The appearance of such ripples on the film surface was noted by the authors of [2, 5], who considered this phenomenon to be the commencement of the turbulent-flow regime. The diffusion of the dye streaks is additional evidence that a change in film-flow regime occurs.

The motion of the surfactant solution at $Re_f^c \geq 60$ and $\omega \geq 200\ sec^{-1}$ is also characterized by the presence of concentric waves in the central portion of the disk and intense liquid mixing at the periphery. Foaming and tear-off of liquid peaks by the air flow occurred, resulting in significant numbers of fine droplets being carried off.

During the experiments with water and the glycerin solution measurements were made of the disk radii R_{cr_1} at which the transition from solitary waves to ripple region and flow mixing occurred. The R_{cr_1} value was taken as the distance from the disk center at which it was no longer possible to distinguish solitary waves. Using these R_{cr_1} values with G and ω, the values of Eqs. (2), (3) were calculated (Table 1). Calculations showed that the values of Re_m obtained with Eq. (2) for water were close to those of [5]; however, for the glycerin solution they differed by a factor of 4-5 times from those calculated for water (approximately the same as the ratio of glycerin solution viscosity to that of water). At the same time, the separate values of Re_m^{cr} for water and glycerin solution under different experimental conditions were close in value, which indicates the validity of the structure of Eq. (2). Calculations with Eq. (3) showed that the values of corrected Reynolds number obtained for water and glycerin solution were practically identical, close to $1 \cdot 10^6$.

Using experimental data from [2, 4, 5, 17],* for conditions close to the present ones, the numbers Re_f and Re_c were calculated (Table 2). Table 2 also shows flow-regime characteristics as specified by the authors of the studies considered. It must be considered that the maximum values of G, ω, and R occurring in the various experiments were used, so that Table 2 shows maximum values attained under these conditions. The data of Table 2 show that the flow-regime characteristics given by the various authors essentially agree with the results of calculation by Eq. (3). An exception is [17], whose authors did not observe turbulent flow, while

*We could not employ [6] because certain data necessary for the calculations are absent.

TABLE 2. Results of Calculations by Eq. (3) for Data of [2, 4, 5, 17]

Reference	Surface form	R_{fin}, m	ω, sec^{-1}	Liquid studied	$Q \cdot 10^6$, m^3/sec	$\nu \cdot 10^6$, m^3/sec	Re_f^c	$Re_c^c \cdot 10^{-6}$ by Eq. (3)	Flow regime as defined by authors of particular study	Flow regime as defined by calc. with Eq. (3), using maximum values of Q, w, and Rfin
[2]	Disk	0.1	360	Water	12	1	19.1	4.6	Turbulent	Turbulent
[4]	Cone	0.074	10.8	Water	26.2	1.308	43.6	1.67	The same	The same
				Glycerin	26.2	33.43	1.28	0.064	Laminar	Laminar
[5]	Disk	0.15	165.5	Water	70	1	74.1	17.8	Turbulent	Turbulent
				Oil	35	23.6	1.57	0.376	Laminar-wave	Laminar-wave
[17]	Cone	0.19	110	Water	13	1	10.8	2.1	Laminar-wave	Turbulent
				Aqueous solution of glycerin	13	2.6	4.15	0.807	The same	Laminar-wave

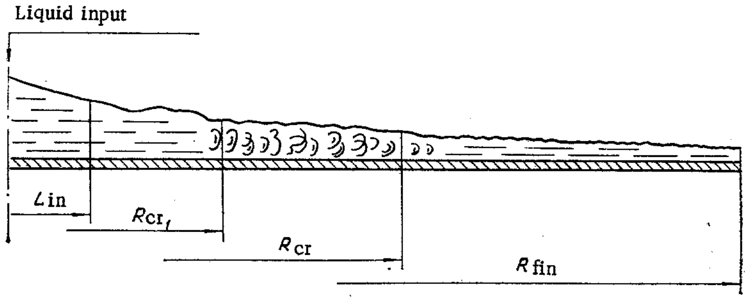

Fig. 2. Hydrodynamic model of film flow on a rotating disk.

Eq. (3) indicates that it should have occurred for water. It seems to us that the authors of [17] were unable to note clearly expressed characteristics of turbulent flow because the maximum flow rates and angular velocities in [17] were 2.5-3 times lower than in the present experiments. Thus, as analysis of the conditions of motion, experimental results, and experimental data of other authors show, Eq. (3) may, in a first approximation, be used for finding the transition from laminar-wave to turbulent film motion.

The results of the experiments performed do not permit an unambiguous conclusion as to whether at $Re_c > 1 \cdot 10^6$ turbulent film flow will exist over the entire Re_c range, since with increase in disk radius the scale of film perturbations decreases due to reduction in film depth. In this case a gradual transition from turbulent flow to laminar with perturbed film surface is possible, as confirmed by experiments on heat exchange in boiling of a water film on rotating disks with radii of 0.075 and 0.15 m [18]. In isothermal experiments it was impossible to determine the moment of such a transition visually. To find the limit of turbulent pulsation damping it has been suggested that the Weber criterion be used,* a quantity which contains the film thickness. This criterion was used by the authors of [19] to determine the stability limit of a gravitation laminar-wave flow. The critical value We = 1 was taken. Considering the known [4, 6] expressions for depth and velocity of a laminar film moving under the influence of centrifugal forces, it may be written in the form

$$We_C = \left[\frac{G^{\frac{5}{3}}\omega^{\frac{2}{3}}}{(2\pi)^{\frac{5}{3}}R^{\frac{4}{3}}\rho^{\frac{2}{3}}\sigma\nu^{\frac{1}{3}}}\right]^{\frac{1}{2}}. \tag{4}$$

To complete the picture of flow characteristics on a rotating disk we must also consider the question of defining the size of the input segment L_{in}, where film dispersal and velocity field stabilization occur. In [17, 20], L_{in} was found from the expression

$$L_{in} = \left[\frac{G^2}{4\pi^2\rho^2\nu\omega}\right]^{\frac{1}{4}}. \tag{5}$$

Calculation with Eq. (5) reveals that for the conditions of the present experiments L_{in} does not exceed $(10\text{-}50)\cdot 10^{-3}$ m, which agrees with results of visual observations.

Considering the above, the hydrodynamic model of flow over a rotating disk may be represented in the manner shown in Fig. 2, where four zones with different flow characteristics are distinguished: input, first laminar-wave, turbulent, and second laminar-wave with fine-scale surface perturbations. The values of L_{in}, R_{cr_1}, and R_{cr} are determined by Eqs. (5), (3), and (4), respectively.

An experimental study of heat exchange in water-vapor formation in the boiling regime on rotating disks [18] supports the applicability of this model.

NOTATION

ω, angular velocity of rotation; R, current radius; R_{fin}, final radius; R_{cr_1}, R_{cr}, radii at which change in mode of film flow occurs; G, Q, mass and volumetric liquid-flow rates; $\Gamma = (G/2\pi R)$, wetting density; μ, dynamic viscosity; ν, kinematic viscosity; σ, surface tension; ρ, liquid density; ν_g, kinematic viscosity of gas; $Re_f = (\Gamma/\nu\rho)$, film Reynolds number; $Re_r = (\omega R^2/\nu)$, $Re_\delta = \omega\delta^2/\nu$, $Re_m = Re_f Re_r$, various expressions for liquid Reynolds number on rotating disk; $Re_c = Re_f Re_g$, liquid-film corrected Reynolds number; $Re_g = (\omega R^2/\nu_g)$, Reynolds number of gas flow on rotating disk; $We_C = [G^{5/3}\omega^{2/3}/(2\pi)^{5/3}R^{4/3}\rho^{2/3}\sigma\nu^{1/3}]^{1/2}$, Weber

*The suggestion for using this criterion for the characteristic of the liquid-film flow regime on a rotating disk was made by our colleague V. G. Rifert.

number for case of liquid film motion under action of centrifugal forces; $L_{in} = [G^2/2\pi^2\rho^2\nu\omega]^{1/4}$, size of input segment, $j = \omega^2 R$, centrifugal acceleration; g, gravitational acceleration.

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TURBULENT FLOWS OF POLYOX SOLUTIONS IN A TUBE WITH LARGE ROUGHNESS OF THE SURFACE

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UDC 532.542.4

Experimental results on the measurement of frictional drag in the flow of polyox solutions with concentration $(5 \cdot 10^{-6}–10^{-3})$ g/cm^3 in tubes of diameter $d = (32 \pm 1)$ mm with different degrees of surface roughness are presented ($R/k_S = 70.8$; 11.1; 7.5; 2.9).

§1. At present, the question about the effect of polymer addition to water flow on the frictional drag during flow along smooth surfaces has been adequately studied experimentally, and this effect can be evaluated not only qualitatively, but also quantitatively [1]. However, in real conditions any surface has some roughness. It was established experimentally in [2-4] that in the case of presence of surface roughness $R/k_S = 14–60$ a reduction of the drag is observed in the transition flow regime of polymer solutions with $k_S v_*/\nu < 100$. As in the flow along smooth surfaces, the reduction in the frictional drag for the values of the tangential frictional stresses exceeds a certain threshold value independent of the state of the surface [2].

A common drawback of all the known experiments for determining the effect of polymer addition on frictional drag of rough surfaces is the absence of parallel determination of the effect of frictional drag resistance

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